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The gauge technique in supersymmetric QED₂

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Abstract. We construct an extension of the gauge technique to two-dimensional supersymmetric gauge theories. This involves a derivation of the spectral representation of a scalar superpropagator in two dimensions. We apply the method to the massive supersymmetric Schwinger model. In the case that the gauge multiplet mass equals twice that of the matter multiplet we determine the spectral functions of the matter propagator.

1. Introduction

Non-perturbative effects determine many of the physical properties of gauge field theories. For example, it is widely accepted that quarks and gluons cannot exist as free particles as a result of non-perturbative effects in QCD (confinement). In supersymmetric gauge theories there are severe restrictions on the content of the theory, e.g. both matter and gauge particles acquire partners of different spins, with properties and interactions prescribed by supersymmetry. An important question is whether the physics of these theories differs in essential ways from ordinary gauge theories. Problems like the presence or absence of confinement in supersymmetric QCD, and the possibility of non-perturbative breaking of supersymmetry itself, motivate the extension and application of non-perturbative methods to supersymmetric gauge theories.

In the continuum formulation of field theories the Dyson–Schwinger equations provide a means to go beyond perturbation theory. These equations express the quantum equations of motion in terms of an infinite set of non-linear integral equations for the Green functions of the theory. Any method of solution of these equations must in some way short-circuit this infinite set to obtain a finite closed set of equations. This is usually done by inserting an ansatz for one of the Green functions. In gauge field theories one of the requirements of such an ansatz is that it respects the gauge invariance of the theory, i.e. that the resulting Green functions satisfy the Ward identities. An approximation that meets this requirement is the so-called gauge technique or Salam–Delbourgo method [1].

In this paper we will extend the gauge technique to the case of supersymmetric gauge theories in two dimensions. Two-dimensional models are useful for trying out field-theoretical methods which will ultimately be applied to realistic models. The model we shall concentrate on is the supersymmetric extension of the massive Schwinger model [2], i.e. supersymmetric quantum electrodynamics in 1 + 1 dimensions (SQED₂) [3, 4].

Let us briefly illustrate the gauge technique as applied to ordinary QED in 3+1 dimensions. This will make clear which properties of supersymmetric gauge theories are crucial as input to the method. In QED the electron propagator $S(p)$ obeys a Dyson–Schwinger equation in which the three-point vertex function $\Gamma_\mu(p, p-k)$ appears in the combination $S(p)\Gamma_\mu(p, p-k)S(p-k)$. The Salam–Delbourgo method approximates this combination in the following way:

$$S(p)\Gamma_\mu(p, p-k)S(p-k) = \int_{-\infty}^{\infty} d\omega \rho(\omega) \frac{1}{\not{p}-\omega} \gamma_\mu \frac{1}{\not{p}-k-\omega}. \quad (1.1)$$

Here $\rho(\omega)$ is the spectral function appearing in the Källén–Lehmann (or spectral) representation of the electron propagator:

$$S(p) = \int_{-\infty}^{\infty} d\omega \frac{\rho(\omega)}{\not{p}-\omega + i\varepsilon \operatorname{sgn}(\omega)}. \quad (1.2)$$

Using equation (1.2) it is easy to show that the approximation (1.1) satisfies the Ward–Takahashi identity

$$k^\mu S(p)\Gamma_\mu(p, p-k)S(p-k) = S(p-k) - S(p). \quad (1.3)$$

Thus the gauge covariance of the method is guaranteed. Substitution of (1.1) and (1.2) into the Dyson–Schwinger equation for the electron propagator leads to a solvable linear integral equation for $\rho(\omega)$ (see [5] for details of the solution). The short-circuit in this case is the ansatz (1.1), since in fact the full three-point function satisfies its own Dyson–Schwinger equation, which contains the full four-point function, etc. The ansatz (1.1) is an approximation in the sense that the structure of the Ward–Takahashi identity (1.3) leaves the transverse part of Γ_μ unspecified. In the infrared limit $k_\mu \rightarrow 0$, the ansatz (1.1) is the exact solution of the differential Ward identity. The method has also been applied to the massless Schwinger model [6]. In that case the transverse part of Γ_μ is restricted by chiral symmetry, and the gauge technique then gives the exact solution of this model.

The natural arena for the generalisation of the gauge technique to supersymmetric gauge theories is superspace. It is clear from the above discussion that this requires in the first place a generalisation of (1.2), i.e. a Källén–Lehmann representation for the superpropagator of the matter multiplet. The generalisation of (1.1) is then straightforward, since the gauge technique ansatz is just a spectrally weighted sum of lowest-order vertex functions. The construction of a spectral representation for the matter superpropagator is the subject of §2 of this paper.

Throughout this paper we use SQED₂ as an example and application of the method. This model is the supersymmetric version of the massive Schwinger model (which was discussed extensively in [7]). We use the massive model as an application to avoid infrared problems associated with the massless scalar fields of SQED₂. The model has two independent mass parameters, since the gauge multiplet can have a gauge-invariant mass term, with mass μ . The model was originally constructed in [3], and its properties were further investigated in [4]. It can be formulated in terms of a complex scalar (matter) superfield $\Phi(x, \theta)$, minimally coupled to the gauge multiplet, which is

described by a Majorana spinor superfield $V_a(x, \theta)$. The local gauge transformations are

$$\Phi' = e^{-ie\Lambda}\Phi \quad \Phi^{*'} = e^{ie\Lambda}\Phi^* \quad V'_a = V_a + i(\gamma_5 D)_a \Lambda \quad (1.4)$$

where $\Lambda(x, \theta)$ is a real scalar superfield. The Lagrangian density

$$\mathcal{L} = \int d^2\theta \left[-\frac{1}{2} \bar{\nabla} \Phi^* \nabla \Phi + m \Phi^* \Phi + \frac{1}{4} \bar{D} V \bar{D} D \bar{D} V + \frac{1}{2} \mu \bar{D} V \bar{D} V \right] \quad (1.5)$$

is invariant under these transformations. The covariant derivative $\nabla \Phi$ is defined as $\nabla_a \Phi \equiv (D + e\gamma_5 V)_a \Phi$. A covariant supersymmetric gauge-fixing term is

$$\mathcal{L}_{\text{GF}} = \int d^2\theta \left[-\frac{1}{4} \alpha \bar{D} \gamma_5 V \bar{D} D \bar{D} \gamma_5 V + \frac{1}{2} \alpha' \mu \bar{D} \gamma_5 V \bar{D} \gamma_5 V \right]. \quad (1.6)$$

The component expansion of the superfields Φ and V_a is presented in the appendix. The gauge multiplet contains, besides the photon A_μ , a physical Majorana fermion λ and a real scalar field M . The gauge-invariant mass term for the bosonic degrees of freedom reads in component form

$$\mathcal{L} \sim \mu M \epsilon^{\mu\nu} \partial_\mu A_\nu. \quad (1.7)$$

The matter multiplet Φ contains, besides the complex fermion field representing the electron, a complex scalar partner with the same charge. Indeed, owing to the $V\Phi^*\partial\Phi$ and $\bar{V}V\Phi^*\Phi$ couplings in (1.5) SQED₂ is quite similar to scalar quantum electrodynamics.

In §3 we will calculate the one-loop corrections to the superpropagators of the Φ and V superfields. This calculation reveals that the theory simplifies in the special case $\mu = 2m$. We find then that there is no mass-renormalisation at the one-loop level, so that the tree-level proportionality of the masses is automatically retained. We will then in §4 apply the gauge technique to this model and obtain within a certain approximation scheme the same special relation between the masses. Section 5 contains some concluding remarks.

2. The spectral representation and the gauge technique

The first step in the construction of the gauge technique ansätze consists of a derivation of the spectral representation for the superpropagator of the matter multiplet. The derivation is given according to the usual procedure [8]. To keep things as simple as possible, we consider the Wightman function W of a neutral scalar superfield Φ :

$$W(x_1, \theta_1; x_2, \theta_2) \equiv \langle 0 | \Phi(x_1, \theta_1) \Phi(x_2, \theta_2) | 0 \rangle. \quad (2.1)$$

The derivation for the Feynman propagator of a complex scalar superfield is analogous, but notationally more involved. Also the extension to four dimensions is trivial.

Spacetime symmetries, such as invariance under supertranslations and Lorentz transformations, put severe restrictions on the form of the spectral representation. Supertranslations, with generators P_μ for translations and Q_a for supersymmetry transformations, can be represented on a scalar superfield Φ in the following way:

$$\Phi(x + a - i\bar{\epsilon}\gamma\theta, \theta + \epsilon) = e^{ia \cdot P + \bar{\epsilon} Q} \Phi(x, \theta) e^{-ia \cdot P - \bar{\epsilon} Q}. \quad (2.2)$$

The vacuum is translation invariant. Therefore, introducing a complete set of momentum eigenstates $|p_n\rangle$ satisfying $P^\mu|p_n\rangle = p_n^\mu|p_n\rangle$, and using equation (2.2) with $\bar{\varepsilon}_a = 0$, we can straightforwardly derive the expression

$$W(x_1, \theta_1; x_2, \theta_2) = \int d^2q \rho(q; \theta_1, \theta_2) e^{-iq \cdot (x_1 - x_2)} \quad (2.3)$$

where the spectral function ρ is defined by

$$\rho(q; \theta_1, \theta_2) = \sum_n \int d^2p_n \langle 0 | \Phi(0, \theta_1) | p_n \rangle \langle p_n | \Phi(0, \theta_2) | 0 \rangle \delta^2(p_n - q). \quad (2.4)$$

From (2.3) it is clear that W is translation invariant:

$$W(x_1, \theta_1; x_2, \theta_2) \equiv W(x_1 - x_2; \theta_1, \theta_2). \quad (2.5)$$

The vacuum is also assumed supersymmetric, i.e. $Q_a|0\rangle = 0$. From this, using (2.2) with $a_\mu = 0$, we obtain the supersymmetry Ward identity

$$W(x_1 - x_2; \theta_1, \theta_2) = W(x_1 - x_2 - i\bar{\varepsilon}\gamma(\theta_1 - \theta_2); \theta_1 + \varepsilon, \theta_2 + \varepsilon). \quad (2.6)$$

This Ward identity imposes a restriction on the spectral function ρ : it has to obey the equation

$$\rho(p; \theta_1, \theta_2) \exp[\bar{\varepsilon}\not{p}(\theta_1 - \theta_2)] = \rho(p; \theta_1 + \varepsilon, \theta_2 + \varepsilon). \quad (2.7)$$

A straightforward expansion of (2.7) in powers of $\bar{\varepsilon}$ (which can be taken to be infinitesimal) results in the partial differential equation

$$[D(-p, \theta_1) + D(p, \theta_2)]\rho(p; \theta_1, \theta_2) = 0 \quad (2.8)$$

where D is the supersymmetry covariant derivative defined in the appendix. The general solution of (2.8) is:

$$\rho(p; \theta_1, \theta_2) = [\alpha(p)^{\frac{1}{2}} \bar{D}D(p, \theta_1) - \beta(p)] \delta_{12} \quad (2.9)$$

where δ_{12} is the δ -function for the Grassmann variables.

Further restrictions on the spectral representation follow from Lorentz invariance. The functions $\alpha(p)$ and $\beta(p)$ are scalar functions of p^2 . Moreover, physical states require $p^2 \geq 0$ and $p_0 = E > 0$. Thus we may write

$$\alpha(p) = \alpha(p^2)\theta(p^2)\theta(p_0) \quad \beta(p) = \beta(p^2)\theta(p^2)\theta(p_0) \quad (2.10)$$

where θ is the Heaviside step function. The Wightman function can now be written as

$$W(x_1 - x_2; \theta_1, \theta_2) = \int_0^\infty dM^2 [\alpha(M^2)^{\frac{1}{2}} \bar{D}D(x_1, \theta_1) - \beta(M^2)] \delta_{12} W(x_1 - x_2; M) \quad (2.11)$$

where

$$W(x_1 - x_2; M) = \int d^2p \theta(p_0) \delta(p^2 - M^2) e^{-ip \cdot (x_1 - x_2)} \quad (2.12)$$

is the Wightman function of an ordinary scalar particle with mass M .

Essentially the same derivation holds for the Feynman propagator Δ of a complex scalar superfield. The result, written in momentum space, is

$$\Delta(p; \theta_1, \theta_2) = \int_0^\infty dM^2 \frac{\alpha(M^2)^{\frac{1}{2}} \bar{D}D(p, \theta_1) - \beta(M^2)}{p^2 - M^2 + i\varepsilon} \delta_{12}. \quad (2.13)$$

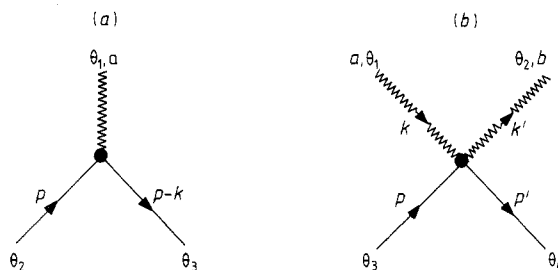


Figure 1. (a) The amputated connected three-point vertex function $\Gamma_a(p, p-k; \theta_1, \theta_2, \theta_3)$. (b) The amputated connected four-point vertex function $G_{ab}(p, k; p', k'; \theta_1, \theta_2, \theta_3, \theta_4)$.

This result can be written more elegantly in terms of a single spectral function $\rho(\omega)$ for $-\infty < \omega < \infty$, the even and odd parts of which serve as independent spectral functions equivalent to α and β in (2.13):

$$\rho(\omega) = |\omega| \alpha(\omega^2) + \text{sgn}(\omega) \beta(\omega^2). \quad (2.14)$$

The final form of the spectral representation then reads

$$\Delta(p; \theta_1, \theta_2) = \int_{-\infty}^{\infty} d\omega \rho(\omega) \frac{\frac{1}{2} \bar{D} D(p, \theta_1) - \omega}{p^2 - \omega^2 + i\epsilon} \delta_{12}. \quad (2.15)$$

The spectral function $\rho(\omega)$ can be normalised according to the sum rule

$$\int_{-\infty}^{\infty} d\omega \rho(\omega) = 1. \quad (2.16)$$

From (2.13) and (2.15) the spectral representations for the propagators of the physical component fields are found to have their usual form, e.g. for the spin- $\frac{1}{2}$ component we reproduce equation (1.2).

The right-hand side of equation (2.15) has the form of a spectrally weighted integral of free propagators with mass ω :

$$\Delta(p; \theta_1, \theta_2) = \int_{-\infty}^{\infty} d\omega \rho(\omega) \Delta^{(0)}(p; \theta_1, \theta_2; \omega). \quad (2.17)$$

The quantity $\Delta^{(0)}(p; \theta_1, \theta_2; m)$ is the bare propagator of a scalar superfield with mass m ; it follows from the Lagrangian density

$$\mathcal{L} = \int d^2\theta \Phi^*(x, \theta) [\frac{1}{2} \bar{D} D + m] \Phi(x, \theta). \quad (2.18)$$

In (2.17) the free propagator is recovered if we take $\rho(\omega) = \delta(\omega - m)$.

The Lagrangian density (1.5) for SQED₂ contains the three- and four-point interaction terms $V\Phi^* \partial \Phi$ and $\bar{V}V\Phi^* \Phi$. Owing to gauge invariance the Green functions corresponding to these interactions obey a set of Ward identities. For the amputated connected three-point vertex function $\Gamma_a(p, p-k; \theta_1, \theta_2, \theta_3)$ (see figure 1(a)) the Ward identity reads

$$\begin{aligned} \bar{D}(k, \theta_1) \gamma_5 \int d^2\theta_4 d^2\theta_5 \Delta(p; \theta_2, \theta_4) \Gamma(p, p-k; \theta_1, \theta_4, \theta_5) \Delta(p-k; \theta_5, \theta_3) \\ = e(\delta_{21} \Delta(p-k; \theta_1, \theta_3) - \Delta(p; \theta_2, \theta_1) \delta_{13}). \end{aligned} \quad (2.19)$$

The gauge-technique approximation for the three-point vertex is a spectrally weighted integral of lowest-order terms:

$$\begin{aligned}
 & \int d^2\theta_4 d^2\theta_5 \Delta(p; \theta_2, \theta_4) \Gamma_a(p, p-k; \theta_1, \theta_4, \theta_5) \Delta(p-k; \theta_5, \theta_3) \\
 &= \int d\omega \rho(\omega) \int d^2\theta_4 d^2\theta_5 \Delta^{(0)}(p; \theta_2, \theta_4; \omega) \\
 & \quad \times \Gamma_a^{(0)}(p, p-k; \theta_1, \theta_4, \theta_5) \Delta^{(0)}(p-k; \theta_5, \theta_3; \omega).
 \end{aligned} \tag{2.20}$$

Using the spectral representation (2.15) and the bare vertex

$$\Gamma^{(0)}(p, p-k; \theta_1, \theta_2, \theta_3) = \frac{1}{2} e [\delta_{13} \gamma_5 D(-p, \theta_1) \delta_{12} - \delta_{12} \gamma_5 D(p-k, \theta_1) \delta_{13}] \tag{2.21}$$

one finds that the approximation (2.20) indeed obeys the Ward identity (2.19). As in ordinary QED the vertex Γ_a is not completely determined by the gauge technique: any function of the form $D_a(k, \theta_1) F(p^2, (p-k)^2; \theta_1, \theta_2, \theta_3)$ can be added to the right-hand side of (2.20) without violating the Ward identity.

A shorthand notation for (2.20), omitting all arguments, is:

$$\Delta \Gamma_a \Delta = \int_{-\infty}^{\infty} d\omega \rho(\omega) \Delta^{(0)}(\omega) \Gamma_a^{(0)} \Delta^{(0)}(\omega). \tag{2.22}$$

An analogous expression can be written down for the amputated connected four-point function $G_{ab}(p, k; p', k'; \theta_1, \theta_2, \theta_3, \theta_4)$ of figure 1(b). In the same notation as (2.22), the Salam–Delbourgo ansatz for this vertex reads:

$$\Delta G_{ab} \Delta = \int_{-\infty}^{\infty} d\omega \rho(\omega) \Delta^{(0)}(\omega) G_{ab}^{(0)} \Delta^{(0)}(\omega). \tag{2.23}$$

Again the spectrally weighted sum of lowest-order vertex functions (2.23) obeys the appropriate Ward identity, as is easily seen if one uses the bare irreducible four-point vertex

$$\Gamma_{ab}^{(0)}(p, k; p', k'; \theta_1, \theta_2, \theta_3, \theta_4) = 2e^2 \delta_{ab} \delta_{12} \delta_{13} \delta_{14}. \tag{2.24}$$

3. One-loop two-point functions

The ultimate aim of the gauge technique, and of any non-perturbative method, is to obtain expressions for Green functions which go beyond the results of simple perturbation theory. It is nevertheless useful to have the perturbative calculations at hand. Therefore, before applying the gauge technique to SQED₂, we will discuss one-loop corrections to the superpropagators of the matter superfield Φ and the gauge

superfield V_a . We find that with a special choice of the two masses, $\mu = 2m$, there is no one-loop correction to the masses.

Recall that the free propagator for the scalar superfield Φ of mass m reads

$$\Delta^{(0)}(p; \theta_1, \theta_2) = \frac{\frac{1}{2}\bar{D}D(p, \theta_1) - m}{p^2 - m^2} \delta_{12} \quad (3.1)$$

and its inverse is

$$\Gamma^{(0)}(p; \theta_1, \theta_2) = [\frac{1}{2}\bar{D}D(p, \theta_1) + m] \delta_{12}. \quad (3.2)$$

For the gauge superfield V_a the propagator and its inverse are:

$$\begin{aligned} \Delta_{ab}^{(0)}(k; \theta_1, \theta_2) = & -\frac{1}{4k^2(k^2 - \mu^2)} [(\not{k} + \mu) (\frac{1}{2}\bar{D}D(k, \theta_1) + \not{k})]_{ab} \delta_{12} \\ & + \frac{1}{4k^2(\alpha^2 k^2 - \alpha'^2 \mu^2)} [(\alpha \not{k} + \alpha' \mu) (\frac{1}{2}\bar{D}D(k, \theta_1) - \not{k})]_{ab} \delta_{12} \end{aligned} \quad (3.3)$$

$$\begin{aligned} \Gamma_{ab}^{(0)}(k; \theta_1, \theta_2) = & [(\frac{1}{2}\bar{D}D(k, \theta_1) + \not{k}) (-\not{k} + \mu)]_{ab} \delta_{12} \\ & + [(\frac{1}{2}\bar{D}D(k, \theta_1) - \not{k}) (\alpha \not{k} - \alpha' \mu)]_{ab} \delta_{12}. \end{aligned} \quad (3.4)$$

Clearly the gauge $\alpha = \alpha' = 1$, the generalisation of the Feynman gauge, is convenient, and will be used from now on. The superpropagator (3.3) then simplifies to

$$\Delta_{ab}^{(0)}(k; \theta_1, \theta_2) = -\frac{1}{2} \frac{(\not{k}(\not{k} + \mu))_{ab}}{k^2(k^2 - \mu^2)} \delta_{12}. \quad (3.5)$$

The Dyson–Schwinger equations for the full propagators are

$$\Delta(p; \theta_1, \theta_2) = \Delta^{(0)}(p; \theta_1, \theta_2) + \int d^2\theta_3 d^2\theta_4 \Delta^{(0)}(p; \theta_1, \theta_3) \Sigma(p; \theta_3, \theta_4) \Delta(p; \theta_4, \theta_2) \quad (3.6)$$

$$\begin{aligned} \Delta_{ab}(k; \theta_1, \theta_2) = & \Delta_{ab}^{(0)}(k; \theta_1, \theta_2) \\ & + \int d^2\theta_3 d^2\theta_4 \Delta_{ac}^{(0)}(k; \theta_1, \theta_3) \Pi_{cd}(k; \theta_3, \theta_4) \Delta_{db}(k; \theta_4, \theta_2) \end{aligned} \quad (3.7)$$

where Σ and Π_{ab} are the self-energy and vacuum polarisation respectively. The one-loop approximation to (3.7) is presented in figure 2. These diagrams can be readily evaluated using the free propagators (3.1) and (3.5) and the vertices (2.21) and (2.24). The result is

$$\Pi_{ab}(k; \theta_1, \theta_2) = -e^2 I(k^2, m^2, m^2) [(\frac{1}{2}\bar{D}D(k, \theta_1) + \not{k}) (-\not{k} + 2m)]_{ab} \delta_{12}. \quad (3.8)$$

The form of the vacuum polarisation is partly determined by gauge invariance, which requires that $(\frac{1}{2}\bar{D}D(k, \theta_1) - \not{k})\Pi(k; \theta_1, \theta_2) = 0$. This transversality condition is indeed

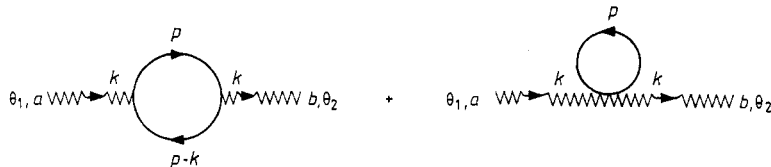


Figure 2. The one-loop vacuum polarisation $\Pi_{ab}(k; \theta_1, \theta_2)$.

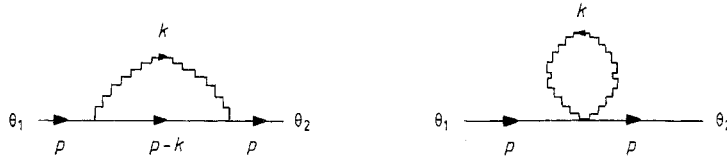


Figure 3. The one-loop self-energy $\Sigma(p; \theta_1, \theta_2)$.

satisfied by (3.8). Therefore only the gauge-independent part of the inverse propagator (3.4) is affected by quantum corrections. The function I in (3.8) is the usual one-loop integral

$$I(p^2, m_1^2, m_2^2) = i \int \frac{d^2 q}{(2\pi)^2} \frac{1}{((p-q)^2 - m_1^2)(q^2 - m_2^2)}. \quad (3.9)$$

Note that for $\mu = 2m$ the vacuum polarisation has the same structure as the gauge-independent part of the inverse bare propagator (3.4), so that to this order in perturbation theory the mass of the gauge multiplet is not renormalised.

For the matter self-energy we have the contributions of figure 3. The supergraph calculation gives

$$\Sigma(p; \theta_1, \theta_2) = -e^2 I(p^2, m^2, \mu^2) (\tfrac{1}{2} \bar{D} D(p, \theta_1) - (m - \mu)) \delta_{12}. \quad (3.10)$$

Note that the contribution of the tadpole diagram in figure 3 vanishes in the Feynman gauge. The result (3.10) corrects an error in reference [4]. The special choice $\mu = 2m$ implies in (3.10) that the one-loop self-energy also has the same form as the inverse propagator, in this case (3.2), so that also the mass of the matter multiplet retains its bare value.

Thus, the choice $\mu = 2m$ has an interesting consequence for SQED₂. It is easy to see that the same phenomenon occurs when an arbitrary number of matter multiplets of mass m are coupled to the gauge multiplet of mass $2m$. We have also verified that the absence of one-loop mass renormalisation holds in arbitrary covariant gauges.

Finally, we note that it is not possible to have either μ or m equal to zero consistently: if one mass is non-zero then the other will be generated by quantum corrections. This is similar to the situation in QED in three dimensions, where a non-zero electron mass generates, through perturbation theory, a gauge-invariant mass for the photon [9].

4. The gauge technique in SQED₂

Armed with the one-loop results of the previous section we are now in a position to apply the gauge technique to the scalar superfield propagator of SQED₂. The standard procedure is to convert the Dyson–Schwinger equation for the matter propagator into a linear integral equation for the spectral function $\rho(\omega)$.

We start from equation (3.6), where both sides have been multiplied by the inverse bare propagator (3.2):

$$(\tfrac{1}{2} \bar{D} D(p, \theta_1) + m) \Delta(p; \theta_1, \theta_2) = \delta_{12} + \int d^2 \theta_3 \Sigma(p; \theta_1, \theta_3) \Delta(p; \theta_3, \theta_2). \quad (4.1)$$

In equation (4.1) $\Sigma(p; \theta_1, \theta_2)$ is the full self-energy of the matter multiplet. The θ -integral on the right-hand side of equation (4.1) is represented diagrammatically in figure 4. To obtain a manageable integral equation for the spectral function a number of approximations must be made. We replace in $\Sigma\Delta$ the full gauge propagators by bare ones, and we also drop the two-loop term containing the full four-point function. The first diagram in figure 4 contains the proper combination of propagators and vertices for the application of the gauge technique. Substitution of the spectral representation (2.15) in the left-hand side, and the Salam–Delbourgo approximation (2.20) in the right-hand side of (4.1) now yields a linear integral equation for $\rho(\omega)$.

It turns out to be convenient to work with the spectral functions $\alpha(\omega^2)$ and $\beta(\omega^2)$ defined in (2.14). The integral equation for $\rho(\omega)$ can then be converted into two coupled linear equations for those spectral functions. On the use of (3.10) with appropriate arguments these equations take the form:

$$\int_0^\infty ds \frac{\beta(s) - m\alpha(s)}{p^2 - s + i\epsilon} = e^2 \int_0^\infty ds \frac{2\beta(s) - \mu\alpha(s)}{p^2 - s + i\epsilon} I(p^2, s, \mu^2) \quad (4.2)$$

$$\int_0^\infty ds \frac{s\alpha(s) - m\beta(s)}{p^2 - s + i\epsilon} = e^2 \int_0^\infty ds \frac{(p^2 + s)\alpha(s) - \mu\beta(s)}{p^2 - s + i\epsilon} I(p^2, s, \mu^2). \quad (4.3)$$

These equations are independent because they follow from the respective coefficients of δ_{12} and $\frac{1}{2}\bar{D}D(p, \theta_1)\delta_{12}$ in equation (4.1). In general the equations (4.2) and (4.3) cannot be decoupled and are therefore difficult to solve. However, there is one exception: for the special choice of masses $\mu = 2m$ the equations simplify to

$$\int_0^\infty ds \frac{\beta(s) - m\alpha(s)}{p^2 - s + i\epsilon} [1 - 2e^2 I(p^2, s, 4m^2)] = 0 \quad (4.4)$$

$$\int_0^\infty ds \frac{(s - m^2)\alpha(s)}{p^2 - s + i\epsilon} [1 - 2e^2 I(p^2, s, 4m^2)] = e^2 \int_0^\infty \alpha(s) I(p^2, s, 4m^2) \quad (4.5)$$

and can now be solved separately.

First consider equation (4.4) and define $\gamma(s) = \beta(s) - m\alpha(s)$. The imaginary part of the equation is

$$\begin{aligned} \pi\theta(p^2)[1 - 2e^2 \operatorname{Re} I(p^2, p^2, 4m^2)]\gamma(p^2) \\ = -\theta(p^2 - 4m^2) \int_0^{M(p^2)} ds \frac{\gamma(s)}{p^2 - s} 2e^2 \operatorname{Im} I(p^2, s, 4m^2), \end{aligned} \quad (4.6)$$

where the upper limit of the integral is given by

$$M(p^2) = (\sqrt{p^2} - 2m)^2. \quad (4.7)$$

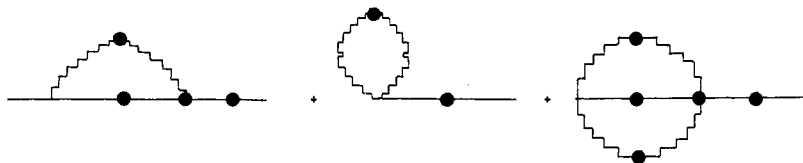


Figure 4. The quantity $\int d^2\theta_3 \Sigma(p; \theta_1, \theta_3) \Delta(p; \theta_3, \theta_2)$. The dots represent connected Green functions.

From equation (4.6) we immediately conclude that $\gamma(p^2)$ vanishes for $0 \leq p^2 < 4m^2$. Direct iteration of equation (4.6) then leads to the result that $\gamma(p^2)$ is identically zero. This is a very important observation: it states that the equality

$$\beta(s) = m\alpha(s) \quad (4.8)$$

is valid beyond perturbation theory. This in turn implies that the full scalar superfield propagator (2.13) can be expressed in terms of only one spectral function $\alpha(s)$:

$$\Delta(p; \theta_1, \theta_2) = (\tfrac{1}{2}\bar{D}D(p, \theta_1) - m)\delta_{12} \int_0^\infty ds \frac{\alpha(s)}{p^2 - s + i\epsilon} \quad (4.9)$$

i.e. the mass of the matter multiplet remains at its bare value.

What remains is to solve the integral equation (4.5) for α , and subsequently to determine the momentum dependence of the propagator (4.9). The method of solution of the α -equation much resembles that of equation (4.4). Introducing the function

$$F(p^2, m^2) = \pi(p^2 - m^2)[2e^2 \operatorname{Re} I(p^2, p^2, 4m^2) - 1] \quad (4.10)$$

which vanishes for $p^2 = m^2$, and the kernel

$$K(p^2, s, m^2) = \frac{e^2}{p^2 - s} (p^2 + s - 2m^2) \operatorname{Im} I(p^2, s, 4m^2) \quad (4.11)$$

we find the following imaginary part of the integral equation (4.5):

$$\theta(p^2)F(p^2, m^2)\alpha(p^2) = \theta(p^2 - 4m^2) \int_0^{M(p^2)} ds K(p^2, s, m^2)\alpha(s). \quad (4.12)$$

For $0 \leq p^2 < 4m^2$ the equation is obviously solved by $\alpha(p^2) = \delta(p^2 - m^2)$ and in fact it is easily seen that this solution extends to $p^2 = 9m^2$. This is used as a starting point for an iterative procedure for finding the solution α for all positive p^2 . The first step determines $\alpha(p^2)$ up to $p^2 = 25m^2$:

$$\alpha(p^2) = \delta(p^2 - m^2) + \theta(p^2 - 9m^2) \frac{K(p^2, m^2, m^2)}{F(p^2, m^2)} \quad 0 \leq p^2 < 25m^2. \quad (4.13)$$

The next step enlarges the momentum region where α is known to $p^2 = 49m^2$, and so on. In this way the formal solution

$$\begin{aligned} \alpha(p^2) = & \delta(p^2 - m^2) + \sum_{n=0}^{\infty} \theta[p^2 - (2n+3)^2m^2] \int_{(2n+1)^2m^2}^{M(p^2)} ds_1 \dots \int_{9m^2}^{M(s_{n-1})} ds_n \\ & \times \frac{K(p^2, s_1, m^2)K(s_1, s_2, m^2) \dots K(s_n, m^2, m^2)}{F(p^2, m^2)F(s_1, m^2) \dots F(s_n, m^2)} \end{aligned} \quad (4.14)$$

is obtained. Equation (4.12) determines this solution only up to an arbitrary multiplicative constant. The function α is usually normalised such that the sum rule

$$\int_0^\infty ds \alpha(s) = 1 \quad (4.15)$$

holds. This is equivalent to performing the wavefunction renormalisation of the scalar superfield Φ .

In principle the propagator of the matter multiplet can now be found by substitution of (4.14) in (4.9). However, equation (4.14) is a series expansion in powers of a complicated function, and it is therefore difficult to obtain an explicit expression for the propagator (4.9). On the other hand, equation (4.14) may be viewed as an expansion in powers of e^2 , so the propagator can be calculated order by order in the coupling constant. We will show explicitly, by expanding (4.14) to order e^2 , how the one-loop result (3.10) is recovered. Substitution of (4.14) expanded to order e^2 into (4.9) yields:

$$\Delta(p; \theta_1, \theta_2) = (\tfrac{1}{2}\bar{D}D(p, \theta_1) - m)\delta_{12} \times \left[\frac{1}{p^2 - m^2 + i\epsilon} + \frac{e^2}{\pi} \int_{9m^2}^{\infty} ds \frac{\text{Im } I(s, m^2, 4m^2)}{(p^2 - s + i\epsilon)(s - m^2)} \right]. \quad (4.16)$$

The dispersion relation

$$I(p^2, m^2, 4m^2) = \frac{1}{\pi} \int_{9m^2}^{\infty} ds \frac{\text{Im } I(s, m^2, 4m^2)}{s - p^2 + i\epsilon} \quad (4.17)$$

can be used to evaluate the integral on the right-hand side of (4.16). This leads to the following expression for the propagator to one-loop order:

$$\Delta(p; \theta_1, \theta_2) = \frac{\tfrac{1}{2}\bar{D}D(p, \theta_1) - m}{p^2 - m^2 + i\epsilon} \delta_{12} (1 + e^2 I(m^2, m^2, 4m^2))(1 - e^2 I(p^2, m^2, 4m^2)). \quad (4.18)$$

A comparison of (4.18) with the self-energy (3.10) reveals that we have indeed recovered one-loop perturbation theory, up to a multiplicative constant which is precisely the one-loop wavefunction renormalisation constant.

Note that the lower limits of integration in the general solution (4.14) correspond to the thresholds for the production of one or more gauge particles. For example, we can see that the process of production of a single gauge particle has a threshold at $p^2 = 9m^2 = (m + 2m)^2$. Higher thresholds can also be read off from (4.14), e.g. two gauge particles can only be produced if the momentum equals at least $p^2 = 25m^2 = (m + 2m + 2m)^2$.

Finally, it is worth noting that transverse corrections to the Salam–Delbourgo vertex approximation are always possible. In some models consistency conditions on the propagators restrict the freedom of adding transverse vertex corrections; in other models, such as the massless Schwinger model, the transverse corrections are dictated by an additional symmetry present in the model [6]. This may play a role in a possible extension of the present work to massless SQED₂.

5. Conclusions

In this paper we have constructed an extension of the gauge technique to supersymmetric theories in two dimensions. We have applied this method to SQED₂, and the extension to non-Abelian theories is straightforward.

The existence of a spectral representation for four-dimensional chiral multiplets can be immediately inferred from § 2. Use of the superspace gauge technique in four

dimensions poses an additional problem due to the infinite number of interaction vertices present in four-dimensional supersymmetric gauge theories.

The application of the superspace gauge technique to SQED₂ has revealed an intriguing property of this model. The choice $\mu = 2m$, where μ and m are the gauge and matter multiplet masses, respectively, is preserved by one-loop quantum corrections, and also by the gauge technique approximation in § 4. We expect that this surprise is due to an additional symmetry of the model which holds only for mass values satisfying this proportionality relation. This aspect of two-dimensional supersymmetric gauge theories is presently under investigation.

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Appendix

Our metric convention is $g_{\mu\nu} = \text{diag}(1, -1)$. The two-dimensional γ -matrices obey $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$. We have $\gamma_5 \equiv \gamma^0 \gamma^1$, so that $\gamma_5^2 = 1$ and $\gamma_\mu \gamma_5 = \varepsilon_{\mu\nu} \gamma^\nu$. For the antisymmetric charge-conjugation matrix C we have $C\gamma_\mu C^{-1} = -\gamma_\mu^T$. Majorana spinors ψ_a satisfy $\bar{\psi}_a = \psi_b C_{ba}$. They obey the Fierz identity

$$\psi_a \bar{\chi}_b = -\frac{1}{2} [\delta_{ab} \bar{\chi} \psi + (\gamma_\mu)_{ab} \bar{\chi} \gamma^\mu \psi + (\gamma_5)_{ab} \bar{\chi} \gamma_5 \psi]. \quad (\text{A.1})$$

Two-dimensional superspace is labelled by the set of coordinates (x^μ, θ_a) . Several types of superfields can be defined on this superspace. Of importance to us are the scalar superfield Φ with component expansion

$$\Phi(x, \theta) = A(x) + \bar{\theta} \psi(x) + \frac{1}{2} \bar{\theta} \theta F(x) \quad (\text{A.2})$$

and the spinor (gauge) superfield

$$V_a(x, \theta) = \eta_a(x) - \frac{1}{2} \theta_a M(x) + \frac{1}{2} i (\gamma_5 \gamma^\mu \theta)_a A_\mu(x) - \frac{1}{2} (\gamma_5 \theta)_a N(x) + \frac{1}{2} \bar{\theta} \theta \xi_a(x). \quad (\text{A.3})$$

The physical matter fields are A and ψ ; F is an auxiliary field. The fields M and $\lambda_a = \xi_a + (\bar{\theta} \eta)_a$ are invariant under the gauge transformations (1.4), while A_μ transforms as usual.

Integration on Grassmann coordinates is normalised as follows:

$$\int d^2 \theta \frac{1}{2} \bar{\theta} \theta = 1. \quad (\text{A.4})$$

The δ -function for these variables is defined by

$$\delta_{12} = \frac{1}{2} (\bar{\theta}_1 - \bar{\theta}_2) (\theta_1 - \theta_2). \quad (\text{A.5})$$

We make frequent use of the supersymmetry covariant derivatives D and \bar{D} . For these we adopt the momentum space representation

$$D_a(p, \theta) = -\frac{\partial}{\partial \bar{\theta}_a} - (\not{p} \theta)_a \quad \bar{D}_a(p, \theta) = \frac{\partial}{\partial \bar{\theta}_a} + (\bar{\theta} \not{p})_a \quad (\text{A.6})$$

with the anticommutation relations:

$$\{D_a(p, \theta), \bar{D}_b(p, \theta)\} = -2\bar{\not{p}}_{ab}. \quad (\text{A.7})$$

Two frequently used identities are

$$\bar{D}(p, \theta)\gamma_\mu D(p, \theta) = -2p_\mu; \quad (\text{A.8})$$

$$\bar{D}(p, \theta)\gamma_5 D(p, \theta) = 0. \quad (\text{A.9})$$

These lead to the simple Fierz identity

$$D_a \bar{D}_b(p, \theta) = -\frac{1}{2}\delta_{ab} \bar{D}D(p, \theta) - \bar{\not{p}}_{ab}. \quad (\text{A.10})$$

In the calculation of Feynman diagram expressions one often encounters a number of D s operating on δ -functions. Explicit expressions for the basic combinations are

$$D_a(p, \theta_2)\delta_{12} = (\theta_1 - \theta_2)_a \exp(\bar{\theta}_1 \not{p} \theta_2) \quad (\text{A.11})$$

$$\bar{D}D(p, \theta_2)\delta_{12} = -2 \exp(\bar{\theta}_1 \not{p} \theta_2) \quad (\text{A.12})$$

$$\bar{D}(k, \theta_2)D(p, \theta_2)\delta_{12} = [-2 + \bar{\theta}_1(\not{p} - \not{k})\theta_2] \exp(\bar{\theta}_1 \not{p} \theta_2). \quad (\text{A.13})$$

They satisfy the symmetry relations

$$D_a(p, \theta_2)\delta_{12} = -D_a(-p, \theta_1)\delta_{12}; \quad (\text{A.14})$$

$$\bar{D}D(p, \theta_2)\delta_{12} = \bar{D}D(-p, \theta_1)\delta_{12}. \quad (\text{A.15})$$

Further important identities are

$$\bar{D}_a(-p, \theta_1)D_b(p, \theta_2)\delta_{12} = -\frac{1}{2}\delta_{ab}\bar{D}D(p, \theta_2)\delta_{12} - \bar{\not{p}}_{ba}\delta_{12} \quad (\text{A.16})$$

$$D_a(p, \theta_2)\bar{D}D(p, \theta_2)\delta_{12} = -2[\not{p}D(p, \theta_2)\delta_{12}]_a \quad (\text{A.17})$$

$$\bar{D}_a(p, \theta_2)\bar{D}D(p, \theta_2)\delta_{12} = 2[\bar{D}(p, \theta_2)\not{p}\delta_{12}]_a \quad (\text{A.18})$$

$$[\bar{D}D(p, \theta_2)]^2\delta_{12} = 4p^2\delta_{12}. \quad (\text{A.19})$$

Finally we offer a set of identities which simplify calculations involving D -algebra and products of δ -functions:

$$\delta_{12}\delta_{12} = 0 \quad (\text{A.20})$$

$$\delta_{12}D_a(p, \theta_2)\delta_{12} = 0 \quad (\text{A.21})$$

$$\delta_{12}\bar{D}_a(-p, \theta_1)D_b(p, \theta_2)\delta_{12} = \delta_{12}\delta_{ab} \quad (\text{A.22})$$

$$\delta_{12}\bar{D}_aD_b(p, \theta_2)\delta_{12} = -\delta_{12}\delta_{ab} \quad (\text{A.23})$$

$$\delta_{12}\bar{D}D(p, \theta_2)\delta_{12} = -2\delta_{12} \quad (\text{A.24})$$

$$\delta_{12}\bar{D}(k, \theta_2)D(p, \theta_2)\delta_{12} = -2\delta_{12} \quad (\text{A.25})$$

$$[\bar{D}_b(k, \theta_2)\delta_{12}][D_a(p, \theta_2)\delta_{12}] = \delta_{12}\delta_{ab} \quad (\text{A.26})$$

$$[D_a(p, \theta_2)\delta_{12}][\bar{D}D(k, \theta_2)\delta_{12}] = -2D_a(p + k, \theta_2)\delta_{12} \quad (\text{A.27})$$

$$[\bar{D}D(k, \theta_2)\delta_{12}][\bar{D}D(p, \theta_2)\delta_{12}] = -2\bar{D}D(p + k, \theta_2)\delta_{12}. \quad (\text{A.28})$$

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